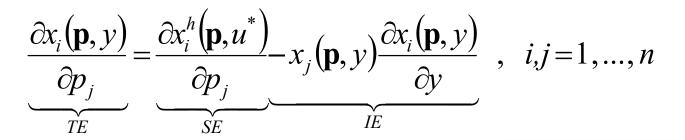


9. The Slutsky Equation





Let **x**(**p**, y) be the consumer's Marshallian demand system. Let u^* be the level of utility the consumer achieves at prices **p** and income y. Then



$$\left(u^* = v(\mathbf{p}, y)\right)$$



Proof

For any prices and level of utility u^* $x_i^h(\mathbf{p}, u^*) = x_i(\mathbf{p}, e(\mathbf{p}, u^*))$

Because this holds for all $\mathbf{p} \gg \mathbf{0}$, we can differentiate both sides with respect to p_j and the equality is preserved. The Hicksian demand on the left-hand side, the Marshallian demand on the right-hand side

 $\frac{\partial x_i^h(\mathbf{p}, u^*)}{\partial p_i} = \frac{\partial x_i(\mathbf{p}, e(\mathbf{p}, u^*))}{\partial p_i} + \frac{\partial x_i(\mathbf{p}, e(\mathbf{p}, u^*))}{\partial v} \frac{\partial e(\mathbf{p}, u^*)}{\partial p_i}$



We know that the minimum expenditure at prices \mathbf{p} and maximum utility that can be achieved at prices \mathbf{p} and income y is equal to income y. Therefore $e(\mathbf{p}, u^*) = e(\mathbf{p}, v(\mathbf{p}, y)) = y$ 2

The partial with respect to P_j of the expenditure function in (1) is just the Hicksian demand for good j at utility u^* . Because $u^* = v(\mathbf{p}, y)$, this must also be the Hicksian demand for good j at utility $v(\mathbf{p}, y)$

$$\frac{\partial e(\mathbf{p}, u^*)}{\partial p_j} = x_j^h(\mathbf{p}, u^*) = x_j^h(\mathbf{p}, v(\mathbf{p}, y)) = x_j(\mathbf{p}, y) \ (3)$$

(by Shaphard's lemma)



To complete the proof, substitute from (2) and (3) into (1) to obtain

$$\frac{\partial x_i^h(\mathbf{p}, u^*)}{\partial p_j} = \frac{\partial x_i(\mathbf{p}, y)}{\partial p_j} + \frac{\partial x_i(\mathbf{p}, y)}{\partial y} x_j(\mathbf{p}, y)$$

With a bit of rearranging, we have what we wanted to show

$$\frac{\partial x_i(\mathbf{p}, y)}{\partial p_j} = \frac{\partial x_i^h(\mathbf{p}, u^*)}{\partial p_j} - x_i(\mathbf{p}, y) \frac{\partial x_i(\mathbf{p}, y)}{\partial y} \quad i, j = 1, \dots, n$$



Slutsky equations provide neat analytical expressions for substitution and income effects. They also give us an 'accounting framework', detailing how these must combine to explain any total effect of a given price change. Yet by themselves, the Slutsky relations do not answer any of the questions we set out to address.

