10. Negative Own-Substitution Terms

Let $x_i^h(\mathbf{p},u)$ *be the Hicksian demand for good i. Then*

$$\frac{\partial x_i^h(\mathbf{p},u)}{\partial p_i} \le 0, \quad i = 1,\ldots,n$$

Proof

The derivative property of the expenditure function, tells us that for any **p** *and u*

$$\frac{\partial e(\mathbf{p},u)}{\partial p_i} = x_i^h(\mathbf{p},u) \quad (\textit{Shepard's lemma})$$

$$0 \geq \frac{\partial^2 e(\mathbf{p},u)}{\partial \mathbf{p}_i^2} = \frac{\partial x_i^h(\mathbf{p},u)}{\partial \mathbf{p}_i}$$

$$\Rightarrow by\ 'e(\mathbf{p},u)\ is\ concave\ in\ \mathbf{p}'$$

Symmetric Substitution Terms

$$\frac{\partial x_i^h(\mathbf{p},u)}{\partial p_j} = \frac{\partial x_j^h(\mathbf{p},u)}{\partial p_i}, \quad i,j = 1,\ldots,n$$

(*proof by Young's theorem*)